Name:

SM275 · Mathematical Methods for Economics　　　　Fall 2019 · Uhan

Quiz 4 – 18 September 2019

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
5	1	
Total		/ 50

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Recall the national income model from Lesson 7, with marginal propensity to consume $m = \frac{1}{2}$ and accelerator $\ell = \frac{1}{6}$:

$$\begin{aligned} T_n &= C_n + I_n + G_n \\ C_{n+1} &= \frac{1}{2} T_n \\ I_{n+1} &= \frac{1}{6}(C_{n+1} - C_n) \\ G_n &= 1 \end{aligned} \qquad n = 0, 1, 2, \ldots$$

where at time n, T_n is the total national income, C_n is the amount of consumer expenditures, I_n is the amount of private investment, and G_n is the amount of government expenditures. We showed that we can rewrite this model as the following DS:

$$T_{n+2} = \frac{7}{12} T_{n+1} - \frac{1}{12} T_n + 1 \qquad n = 0, 1, 2, \ldots \qquad (\star)$$

Problem 1. Find the general solution to the DS $(\star)$.

Problem 2. Suppose $C_0 = 4$ and $I_0 = 5$. Find the IC for the DS $(\star)$.

For Problems 3, 4 and 5, consider the following DS:

$$A_{n+2} = \frac{1}{2}A_{n+1} - \frac{3}{64}A_n + 35 \qquad n = 0, 1, 2, \ldots \tag{$\star\star$}$$

The general solution to the DS ($\star\star$) is

$$A_n = c_1\left(\frac{1}{8}\right)^n + c_2\left(\frac{3}{8}\right)^n + 64.$$

Problem 3. Show that the fixed point of the DS ($\star\star$) is 64.

Problem 4. Is the system stable, unstable, or neither? Briefly explain.

Problem 5. Is the fixed point 64 attracting, repelling, or neither? Briefly explain.

The general solution of the **second order linear DS** $A_{n+2} = aA_{n+1} + bA_n + c$, $n = 0, 1, 2, \ldots$ is

$$A_n = c_1 r^n + c_2 s^n + \frac{c}{1-a-b} \qquad \text{if } a + b \neq 1 \text{ and } r \neq s$$

$$A_n = (c_1 + c_2 n)r^n + \frac{c}{1-a-b} \qquad \text{if } a + b \neq 1 \text{ and } r = s$$

$$A_n = c_1(a-1)^n + c_2 + \frac{c}{2-a}n \qquad \text{if } a + b = 1 \text{ and } a \neq 2$$

$$A_n = c_1 + c_2 n + \frac{c}{2}n^2 \qquad \text{if } a + b = 1 \text{ and } a = 2, b = -1$$

where r and s are the roots of the characteristic equation $x^2 = ax + b$.